Knot Physics: Vacuum Geometry - DRAFT

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Abstract

In a previous work, we describe a branched four-dimensional spacetime manifold embedded in a six-dimensional Minkowski space. In this paper, we provide additional information about the geometry of the vacuum. In this description, the classical vacuum can be described as Lorentz invariant, and the quantum vacuum is best described as Lorentz isotropic. We provide evidence that the vacuum has properties corresponding to a vacuum energy and a vacuum temperature.

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1 Introduction

We assume that the spacetime manifold M is a branched manifold embedded in a Minkowski 6-space as described in “Physics on a Branched Knotted Spacetime Manifold” [1]. On M there is a metric $g_{\mu\nu}$ that is Ricci flat, written $\hat{R}^{\mu\nu} = 0$. We claim that these constraints allow M to have a vacuum with branches that are able to spontaneously split and recombine. To match physical observations, this vacuum should also be Lorentz isotropic. In this paper we describe how this branching can be Ricci flat and how it can be Lorentz isotropic.

2 Boundary conditions of branch separation

Assume that B_1 and B_2 are branches of M that are separate from each other only within a bounded region A. Let A_1 and A_2 be the regions of B_1 and B_2 where the branches are separate. Then A_1 and A_2 have the same boundary as in Figure 1. On that boundary, the metric $g_{\mu\nu}$ must be single-valued because a branched manifold must have a unique tangent space at every point. We claim that A_1 and A_2 can be distinct from each other despite the constraint that $g_{\mu\nu}$ is everywhere Ricci flat.

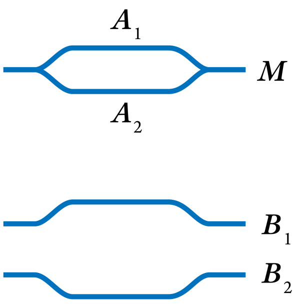

Figure 1: The branched manifold M consists of branches B_1 and B_2 which are separate from each other in the regions labeled A_1 and A_2.

On a topologically trivial manifold, the Ricci flatness constraint and the boundary condition would uniquely determine the value of $g_{\mu\nu}$ everywhere. For that reason, we look at topological defects to find degrees of freedom for the branches.

As shown in previous work, the constraints on M allow the production of pairs of topological defects of homeomorphism class $\mathbb{R}^3\#(S^1 \times P^2)$. To maintain Ricci flatness, this type of topological defect requires that the branch weight $w = (-\det(g))^{1/2}$ scales with distance from the defect according to the approximate relation $w \approx e^{1/r}$ for large r as shown in Figure 2. The gradient of the branch weight ∇w appears in derivatives of $g_{\mu\nu}$ and the gradient therefore must be consistent between the two branches wherever the branches meet. To allow two branches to have consistency of ∇w, we consider topological defects that have the same location but different amplitude.

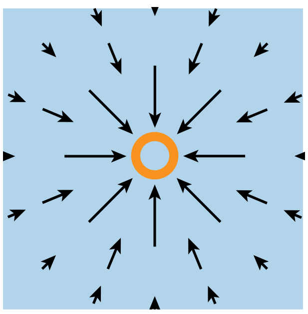

Figure 2: A topological defect $\mathbb{R}^3\#(S^1 \times P^2)$ is represented by the orange circle at the center. The arrows indicate the gradient ∇w.

3 Amplitude splitting

The geometry of an elementary fermion can be described using a map with parameters ξ and θ. The map is from $\mathbb{R}^3 - T$, a 3-space in toroidal coordinates (τ, σ, ϕ) with $\tau > 1$ removed. The map embeds in $\mathbb{R}^5$ with the first three coordinates toroidal and the last two coordinates Cartesian.

$$X(\xi, \theta; \tau, \sigma, \phi) = \left(\frac{\tau}{1-\tau}, \sigma, \phi, \xi\tau\sin(2\sigma+\theta), \xi\tau\cos(2\sigma+\theta)\right). \quad (1)$$

We can make two distinct branches that have fermions with the same location but different amplitudes, $X(\xi_1, \theta; \tau, \sigma, \phi)$ and $X(\xi_2, \theta; \tau, \sigma, \phi)$. If these fermions are at the same location, then the metric $g_{\mu\nu}$ is the same for both of them at large distance. This allows us to have branches that are geometrically distinct but with matching boundary conditions.

If we have a topological defect on one branch B of M, then this amplitude splitting allows us to split B into two branches B_1 and B_2 corresponding to two values of the amplitude, ξ_1 and ξ_2 as in Figure 3.

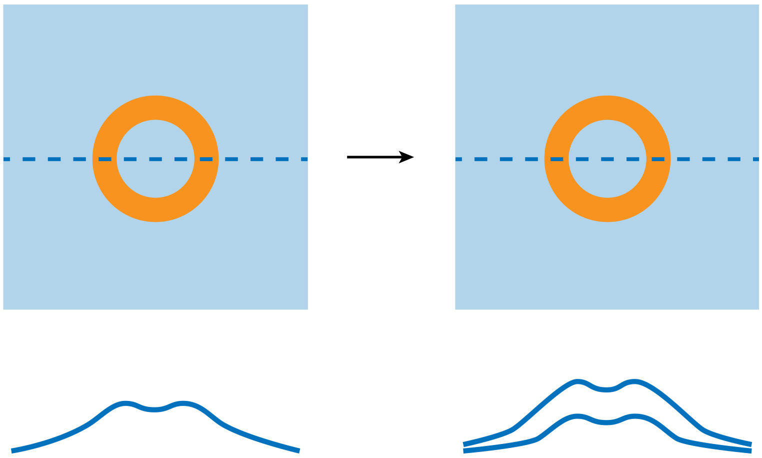

Figure 3: On the top left, we show a constant x^2x^3 slice of M that passes through a topological defect with amplitude ξ. On the bottom left, we show that slice in x_1 and x_4. On the right, we show the same slice after amplitude splitting results in two distinct branches corresponding to two different amplitudes ξ_1 and ξ_2 of the defect.

To split a branch that has no topological defect we first create a pair of topological defects and then perform the amplitude splitting operation on those defects as in Figure 4. In the vacuum, this process consists of creating a virtual particle pair and using the geometry of those virtual particles to split the branch.

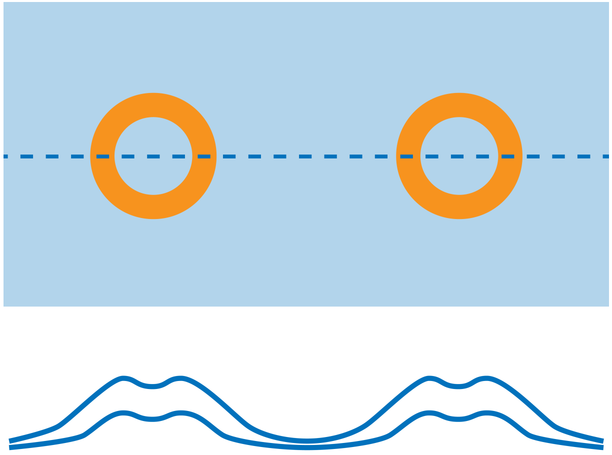

Figure 4: On the top, we show a slice through a virtual particle pair. On the bottom, we show that this pair have amplitude splitting that results in two distinct branches.

4 Lorentz isotropy

In Knot Physics, there is entropy associated with branch splitting and recombination even in the vacuum. From the previous section, we see that the production of virtual particles allows branch splitting and recombination using the amplitude splitting operation. To match physical observation, the vacuum needs to be Lorentz invariant at the classical level. In Knot Physics, the spacetime manifold is not uniform at the quantum level and the branches of the spacetime manifold do not represent every possible quantum state. Instead, the branches of M represent a sample of all possible states. We do not require that the Knot Physics vacuum is Lorentz invariant, but we do require that it is Lorentz isotropic; the branches of M must look the same *on average* in any inertial reference frame. This implies that the distribution of virtual particles of M must be Lorentz isotropic.

If the virtual fermions travel at any velocity other than light speed, $c = 1$, then there will be an average velocity of the virtual fermions that will transform like a 4-vector and the virtual particle distribution will not be Lorentz isotropic. For that reason, it must be that virtual fermions travel at the velocity of light. This is obviously different from the behavior of real fermions and we describe how this is possible in the next section.

5 Virtual fermions travel at c

Real fermions travel at less than c. A real fermion has one knot on every branch of the spacetime manifold. The presence of the real fermion impairs the entropy of the spacetime manifold and the energy-momentum tensor of the fermion comes from that entropy impairment. The energy-momentum tensor of a real fermion is a 4-vector; it becomes infinite at the velocity of light and cannot transform to any velocity faster than light.

The manifold M is constrained such that there is a 3+1 tangent space at every point. A topological defect on M can travel at c while still satisfying the constraint of a 3+1 tangent space as in Figure 5. For that reason, a virtual fermion can travel at light speed. Because there is no energy or momentum associated with a virtual fermion, there is no energy-momentum tensor to become infinite.

Figure 5: We show a geometric feature on a branch of M that is moving with velocity v. Even if $v = c = 1$ this manifold has a 3+1 tangent space at every point.

6 Branch recombination from annihilation

We showed the branch separation that can occur as a result of the creation of a pair of virtual fermions followed by amplitude splitting of those fermions. If those virtual fermions meet virtual photons then they may reverse direction and annihilate. If this happens then the amplitude splitting of the fermions is eliminated, which forces the branches to recombine. The branch recombination propagates on the light cone. In this way, branch separation and recombination are generated by creation and annihilation events of virtual fermions and propagate on light cones, as shown in Figure 6. It is also possible for the virtual fermions to annihilate with virtual fermions that originated in a different pair creation, which would result in more complicated branch structure, as shown in Figure 7. If the distribution of creation, annihilation, and direction changing events is Lorentz isotropic then the system of virtual fermions is Lorentz isotropic. In this way, the vacuum can be Lorentz isotropic.

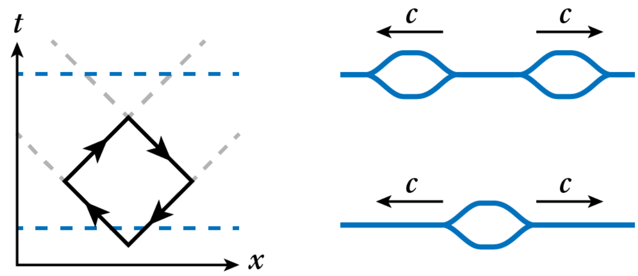

Figure 6: On the left we show the creation and annihilation of a virtual fermion pair. We take constant time slices through the branched spacetime manifold that are indicated by the blue dotted lines. On the right, we show the branching associated with the constant time slices. The first slice shows a branch separation that propagates outward at c. The second slice shows a branch separation followed by branch recombination that propagates outward at c.

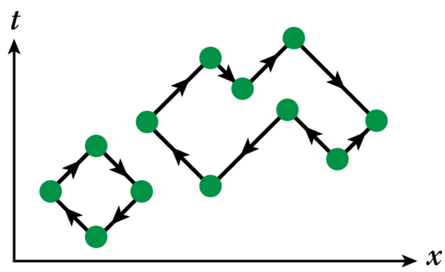

Figure 7: We show the simplest type of creation/annihilation event such that a virtual pair is created and then annihilate with each other. We also show a more complicated case of virtual fermion creation and annihilation that generates a more complicated branch structure. Creation, annihilation, and direction changing events are indicated with green dots.

7 Vacuum temperature and vacuum energy

In "Physics on a Branched Knotted Spacetime Manifold" [1], we showed that the creation of particle/antiparticle pairs requires a strong electric field that allows the distance between the pair to go to zero as measured by $g_{\mu\nu}$.

Assume initial conditions such that M consists of a single branch with no electromagnetic field and no virtual or real particles. Then the constraint that $g_{\mu\nu}$ is Ricci flat implies that there is no opportunity to create virtual fermions. Because fermions are necessary for branching, the single branch of M can never split. In this case we say that the *vacuum temperature* of M is zero.

If we assume initial conditions such that M does have virtual particles and branches, then those virtual particles will annihilate when they collide and that annihilation will result in virtual fields that allow the creation of other virtual particles. It remains to be proven, but it seems likely that there is a conserved *vacuum energy* that corresponds to the rate of production of virtual particles. That vacuum energy would likewise have a vacuum temperature that corresponds to the rate of production of virtual particles per unit of branch weight. In this description, the vacuum energy is equal to the branch weight multiplied by the vacuum temperature.

References

1. Ellgen, C. & Biehle, G. Physics on a Branched Knotted Spacetime Manifold. Unpublished (2022).