Knot Physics: Entanglement and Locality

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Abstract

We describe entanglement and locality in knot physics. In knot physics, spacetime is a branched manifold. The quantum information of a system is encoded in the branches of the manifold. We show how that quantum information can persist despite the continual recombination of the branches of the manifold. We also note that the quantum collapse of state of the branches is non-local. That non-locality allows for non-local effects of entanglement without additional assumptions. We apply this description to the EPR paradox.

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I. INTRODUCTION

This paper describes entanglement in knot physics. Most of the discussion will be understandable as a description of the properties of a branched manifold. The fundamental assumptions of the theory are listed in the Appendix of this paper. For additional background, please see the paper "Physics on a Branched Knotted Spacetime Manifold" [1].

An entangled system is one whose quantum state cannot be factored as product states of its local constituents. The entanglement can persist even if the constituents are physically separated from each other. If the parts of the system are measured, the measurements of the parts are correlated, despite the fact that the states are not definite before the measurement and the measurements may occur at spacelike separation.

Knot physics assumes a branched 4-manifold embedded in a 6-dimensional Minkowski space. Quantum mechanics is a consequence of the interactions of the branches of the manifold. In this paper we describe how those branch interactions produce quantum entanglement. The branch structure of the spacetime manifold contains the quantum information of the system. If we were able to see all the branches of the spacetime manifold at a particular time, we would have all of the quantum information at that time. This means that we would know the entanglement of all the particles without needing to know their history.

The quantum information of the branched manifold can be lost as a result of branch recombination. We find that the dimension of the spacetime manifold has a significant effect on branch recombination. Likewise, the number of possible quantum states has an important effect on branch recombination. We show how both of these factors, manifold dimension and number of quantum states, contributes to the probability of preserving entanglement.

Because the branch structure is non-local, the quantum information is also non-local. Collapse of state occurs by branch recombination, which can occur outside of the causal cone. We show how this allows entanglement on the branched manifold to produce correlated measurements, even if the measurements are performed at spacelike separation.

II. BRANCHING

In this section we describe the branching of the manifold and the way that it applies to quantum mechanics. In Fig. 1 we begin with the simplest non-trivial branched manifold, a branched 1+1-dimensional manifold Y in which the branches separate only once. In Fig. 2 we show how Y can be decomposed into two branches, B_1 and B_2. We define a branch of the manifold to have no boundary, which means that B_1 and B_2 both extend without boundary. The two branches, B_1 and B_2, are identical everywhere except within a bounded region, and are separate on the interior of that region.

A particle corresponds to knots on the spacetime manifold. One real particle has one knot on every branch of the manifold. We see the simplest example of this in Fig. 3, with a single real particle on an unbranched 1+1-manifold. We see a slightly more complex example in Fig. 4, with a single real particle on the branched 1+1-manifold Y. The branches begin together, then separate, and then recombine. The particle likewise begins as a single knot, which then separates into two knots on the separated branches, and then later recombines to a single knot after the branches recombine.

We briefly indicate how this branch structure relates to the quantum wave function. In Fig. 5 we see a particle on a branched manifold that branches once and a branched manifold that branches many times. In both cases the real particle has one knot on each branch. In the case that the manifold branches many times, there are so many knots that we describe them using a continuous distribution. In [1] we show how that distribution corresponds to the complex wave function ψ. In the rest of the paper, we use the branched description of quantum mechanics, as this is the structure we use to describe entanglement.

The branched spacetime manifold is underconstrained, and its dynamics are determined by maximization of entropy. The probability of an event is proportional to the number of branches on which that event occurs. Frequent recombination of branches increases the number of branches. Branches can recombine more frequently if they are close to each other. For this reason, maximization of entropy implies that branches tend to stay close to each other. We call this tendency to stay close to each other "branch cohesion". In Fig. 6 we see an example of branch cohesion, where branches staying close to each other increases the number of branches.

In Fig. 7 we see the relationship between collapse of state and branch cohesion for the relatively simple case of a 0+1-manifold. If each branch must follow one of two possible paths, and those paths become increasingly different from each other, then the branches will collectively collapse to one of the paths so that the branches can stay close to each other. As we will see in this paper, increasing the dimension of the branched manifold increases the complexity of collapse of state.

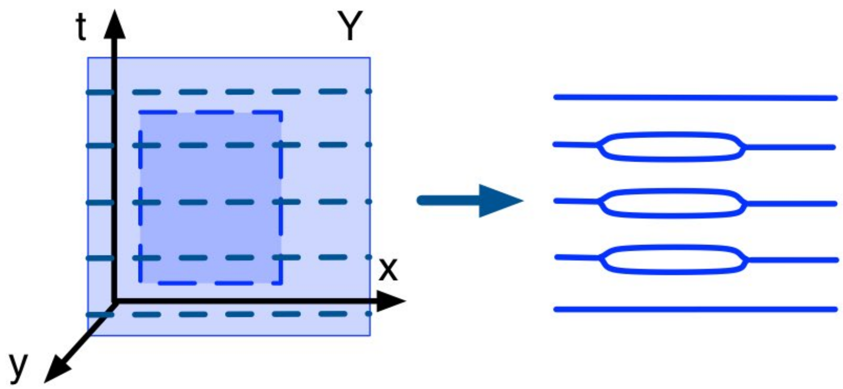

FIG. 1 – We assume a branched spacetime manifold. In the diagram, we see a branched 1+1-manifold Y embedded in a Minkowski 2+1-space. The manifold branches once, such that the two branches are identical on the exterior of the dotted rectangle and separate on the interior of the dotted rectangle. We take a few constant time slices to show how the branches separate. In the first slice, there is no branch separation and the constant time slice is just a single line. In the next three slices, the branches are separate within the rectangle. Each of those slices consists of branched 1-manifolds with branches that separate in the middle.

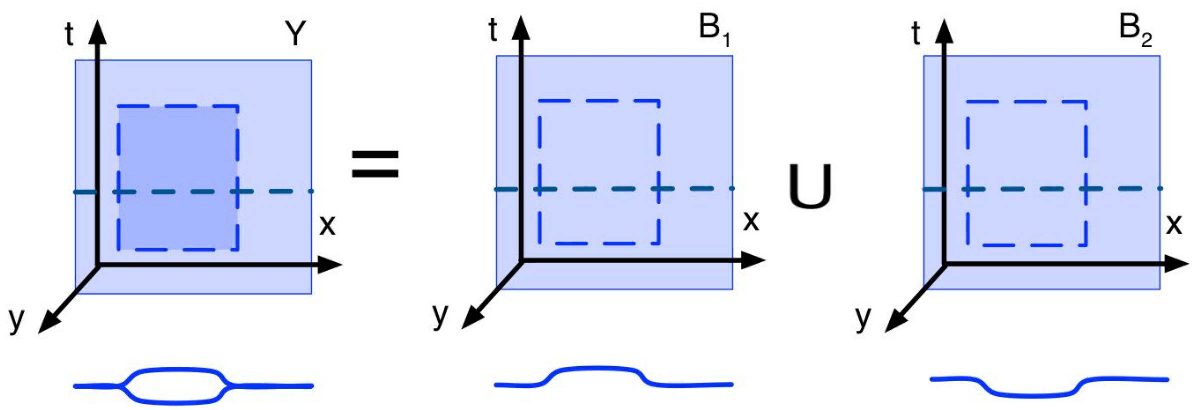

FIG. 2 – The branched manifold Y consists of constituent branches B_1 and B_2, which is to say $Y = B_1 \cup B_2$. In this figure, and in general, we say that the branches of the manifold have no boundary, but rather extend to infinity. We take constant time slices through B_1, B_2, and Y. Each slice is shown below the respective manifold. B_1 and B_2 are unbranched, and their slices are unbranched 1-manifolds. Y is branched and its slice is a branched 1-manifold.

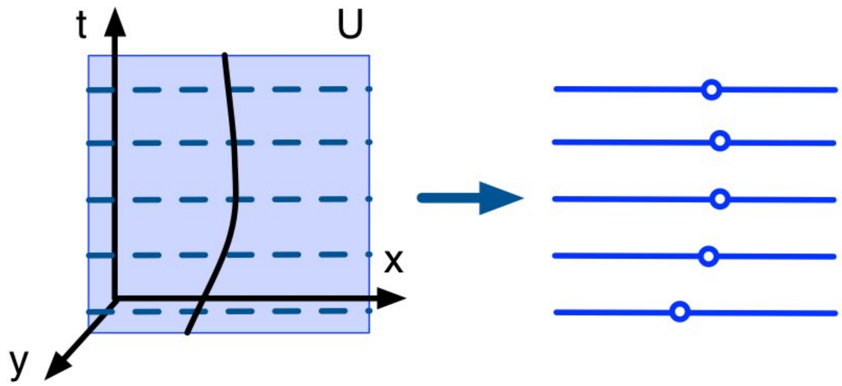

FIG. 3 – We introduce an unbranched 1+1-manifold U that has a single particle. The path of the particle is indicated by the black line. We take slices through U, shown on the right. The particle on U is a knot. On each slice, the knot is indicated by the circle.

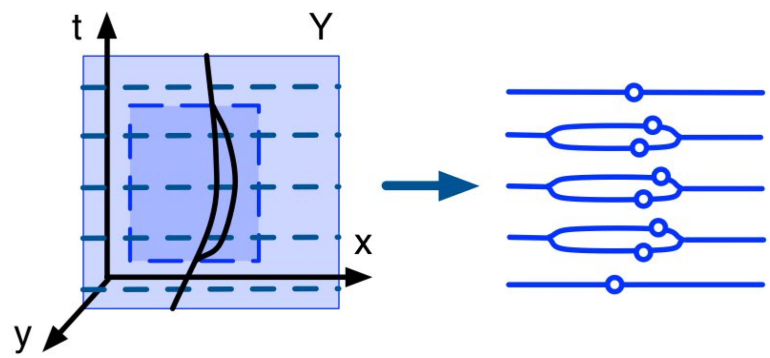

FIG. 4 – In Fig. 3 we showed a particle on an unbranched manifold. We now introduce a particle on the branched 1+1-manifold Y. The particle corresponds to knots on both of the branches, and the paths taken by those knots are shown by the dark lines. We take constant time slices to show how the knots (represented by circles) appear on each of the branches. In the first slice, the branches are not separated, so there is just one knot corresponding to the particle. In the next three slices, the branches are separated, and both of the branches have one instance of the knot. Because the branches are separated, the knots are independent of each other and can move independently. In the last slice, the branches have recombined. The branches must recombine to a single consistent geometry, and this forces the knots to recombine. Thus, in the last slice we again see just one knot.

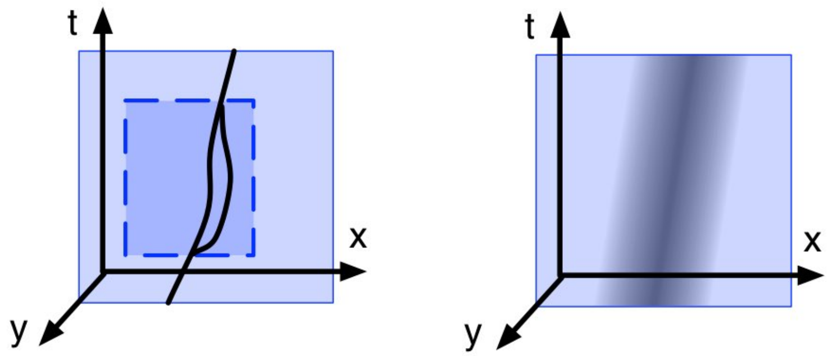

FIG. 5 – In the left diagram we see a particle on a manifold that branches once. In the right diagram we see a particle on a manifold that branches many times. We suppress the representation of the branch separations. There are so many knots that we describe their location using a probability distribution. In [1] we show how the distribution of knots is equivalent to the wave function ψ.

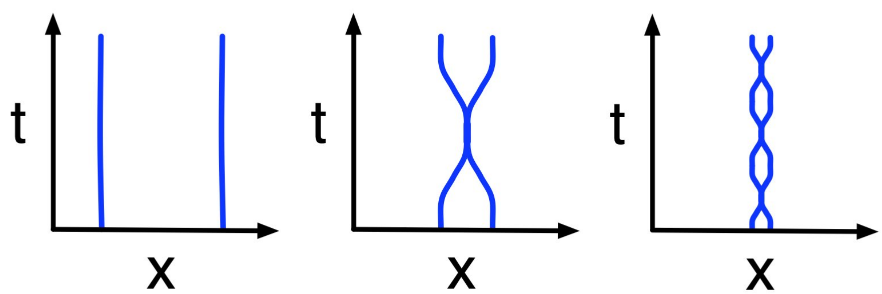

FIG. 6 – These are three diagrams of branched 0+1-manifolds. The diagrams show branched manifolds with the same total branch weight but different entropy. (For an explanation of branch weight, please see [1].) On the left there are just two branches. In the middle diagram we reduce the x-distance between the branches, which allows them to recombine. As a result, there are four branches. The first branch is left-left: it begins on the left and then goes left after the recombination. The other three branches are left-right, right-left, and right-right. The diagram on the right shows branches that maintain small x-distances, which allows frequent recombination and many branches. The large number of branches implies that an event that corresponds to the diagram on the right has greater probability than an event that occurs on the diagram on the left. To maximize entropy, the branched manifold keeps the branches close.

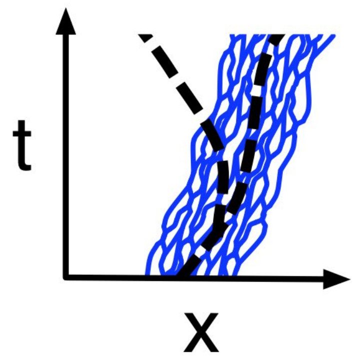

FIG. 7 – The diagram shows collapse of state for a branched 0+1-manifold. We begin with a branched 0+1-manifold and introduce a hypothetical constraint that each branch will stay close to one of two paths, illustrated by the dashed black line that splits into two. The branches will tend to stay close to each other, by branch cohesion. Because the paths diverge beyond the diameter of branch cohesion, the branches stay close to just one of the paths. We say that the state of the manifold has collapsed to one of the paths.

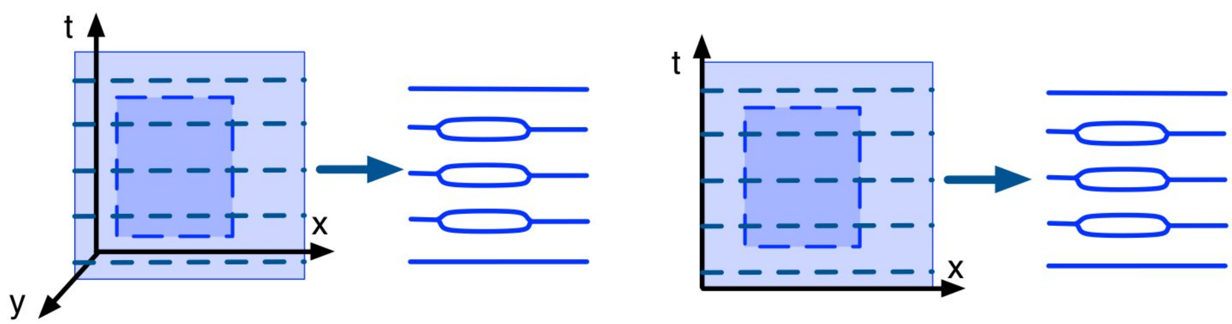

FIG. 8 – In the rest of the paper we will suppress the dimensions of the Minkowski embedding space, for simplicity. For example, the diagram on the left shows the y-dimension that is perpendicular to the spacetime manifold. In the following diagrams we will use the representation on the right, which does not include the y-dimension.

III. EPR PARADOX

We examine the EPR paradox in this theory by considering the particular experiment associated with Bell's inequality. A particle/antiparticle pair are created and separated. Then their spins are measured, at spacelike physical separation. While the outcome of the spin measurement appears to be determined at the time of measurement, the measurements produce opposite spins without the opportunity for any causal information to travel from one measurement to the other.

We show how the EPR experiment occurs on a branched manifold. First, we show how measurement affects the particle state and the branch structure of the manifold. Then we show how manifold dimension and the number of quantum states affect the preservation of quantum information and entanglement.

A. Measurement

To show the EPR experiment on a branched manifold, we begin by showing the measurement of a quantum state on a branched manifold. In Fig. 9, we show a particle passing through a machine that measures its quantum state. We look at time slices of the experiment to show the branches of the manifold both before and after the measurement. On each branch, we see the a copy of the machine that performs the measurement. Naturally, the machine consists of particles and each of those particles has one knot on every branch of the manifold. We also see the particle that is being measured. On each branch of the manifold, there is one knot corresponding to that particle. Before the measurement, the particle corresponds to knots that are in many different states. The state of the machine on each branch before the measurement is presumably complicated, but it is uncorrelated to the state of the knot on its branch. After the measurement, the measurement state of the machine must be the same as the state of the knot on its branch. Furthermore, the machine is macroscopic, meaning that the difference between possible measurement states of the machine is larger than the diameter of branch cohesion. For that reason, the machine collapses its quantum state and the measurement state of the machine must be the same on every branch. In other words, if the machine measurement results in "spin up" on one branch then it must result in "spin up" on every branch. The state of each knot after measurement must be the same as the measurement state of the machine on its branch, and therefore each knot must be in the same state after measurement, for example, "spin up". In Fig. 10, we show the same experiment, but this time we simplify the branch structure so that we can emphasize the way that quantum information is associated with the branch structure of the manifold. In this simplified diagram, we take time slices and label each point on the branches according to what type of branch it is on. On branch B_1, the knot has spin up before the measurement. On branch B_2, the knot has spin down before the measurement. Each point on the branches is labeled according to whether it is on B_1, B_2, or $B_1 \cap B_2$. In this way we label each point according to the quantum states to which it is "connected". We will use this notion of connection by branches to describe entanglement in this theory. In Fig. 11 we combine the complexity of Fig. 9 with the labeling of Fig. 10. If the outcome of the measurement corresponds to B_1, then all of the points on the branches after measurement must be connected to the initial B_1 state. For that reason, all of the points after measurement are of type B_1 or $B_1 \cap B_2$.

In Fig. 12 we show the EPR experiment. We create a particle/antiparticle pair and then measure the spins. In Fig. 13 we see how the experiment appears on a branched 1+1-manifold. The particle and antiparticle have knots on each of the branches. The knots have spins (for more about the knot geometry of particle spin, see [1]). On each branch, the spins of the particle and the antiparticle are opposite to each other. When the branches recombine, the individual spins of the knots on each branch must recombine to a single spin state. In Fig. 14 we see the same experiment performed, but the branches recombine in the middle. That recombination in the middle implies that the quantum states of the particle and antiparticle are in a product state. In this example, the entanglement is lost.

We have seen an example of entanglement being preserved (Fig. 13) and entanglement being lost (Fig. 14). In the following sections, we will examine factors that affect the probability that entanglement will be preserved.

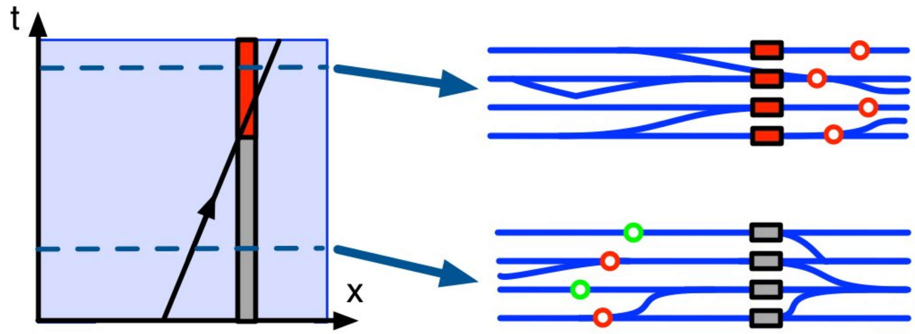

FIG. 9 – On the left, we show a particle passing through a machine. The machine is represented by the rectangle. The machine measures the state of the particle and inflates the result of that measurement to macroscopic size. The result of the measurement is represented by the color of the machine, in this case red. On the right, we show time slices of the experiment before and after the measurement. We show the branches of the manifold and the knots corresponding to the particle. The color of each circle indicates the state of the knot. On each branch, there is a rectangle representing the machine. In the first slice, the rectangles are gray, indicating that they are uncorrelated to the particle state. The particle passes through the machine and interacts with it. The machine is designed so that the interaction produces a macroscopic effect, a measurement of the particle state. The macroscopic effect is larger than the diameter of branch cohesion (see Fig. 7), and so the state measured by the machine must be the same on every branch (in this case red). After measurement, the state of each knot must be the same as the state of the machine on its branch. This forces all the knots to likewise be red.

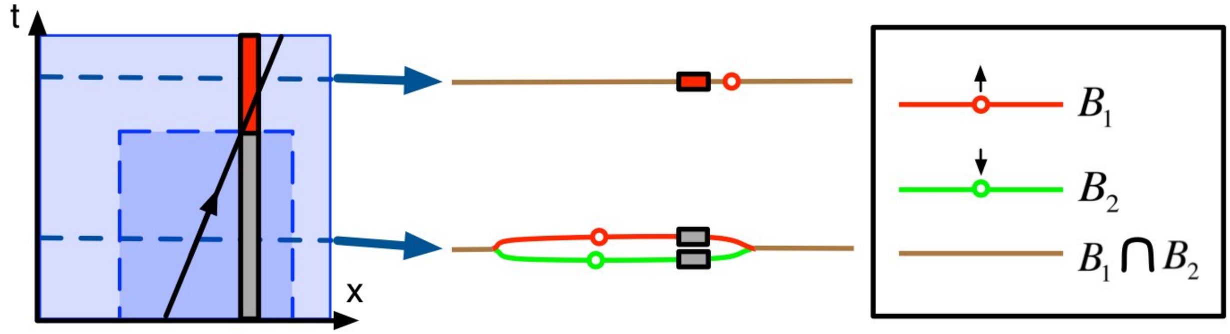

FIG. 10 – We perform the same experiment as Fig. 9. In this diagram we simplify the branch structure so that we can emphasize the way that quantum information is associated with the branches of the manifold. On branch B_1, the knot initially has spin up. On branch B_2, the knot initially has spin down. We color the points on the branches according to three types. Points on B_1 are labeled red. Points on B_2 are labeled green. Outside of a bounded region, every point on the manifold is on both branches. These points are of type $B_1 \cap B_2$, labeled brown. In this example, the machine measures the state of the particle and the state collapses in the simplest way possible: the branches recombine and the resulting knot has one of the two initial states, in this case red.

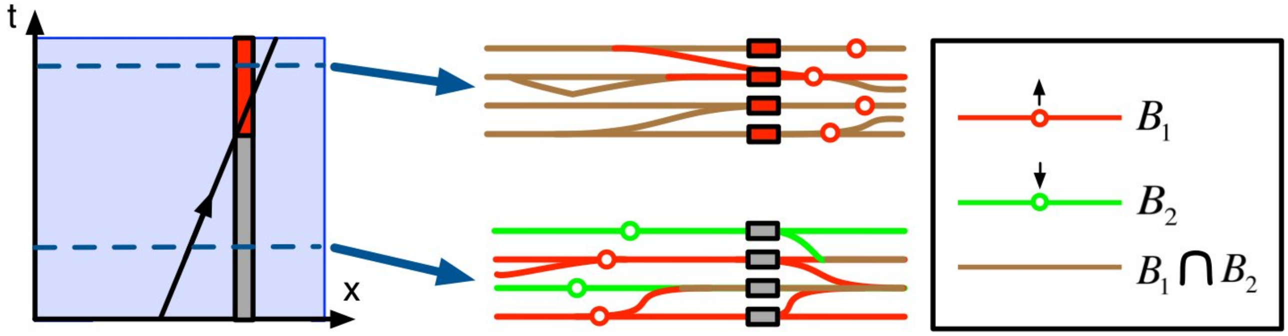

FIG. 11 – We perform the same experiment as Fig. 9 and Fig. 10. In this diagram we show the experiment with more complicated branch structure. Points of type B_1 (labeled red) are on a branch such that the knot initially has spin up. Points of type B_2 (labeled green) are on a branch such that the knot initially has spin down. Points that are on a branch connected to both initial states are of type $B_1 \cap B_2$ (labeled brown). The machine measures the state to be red, forcing the knots to be red. After measurement, the points are all connected to the initial red state, and therefore all points are of type B_1 or $B_1 \cap B_2$ after measurement.

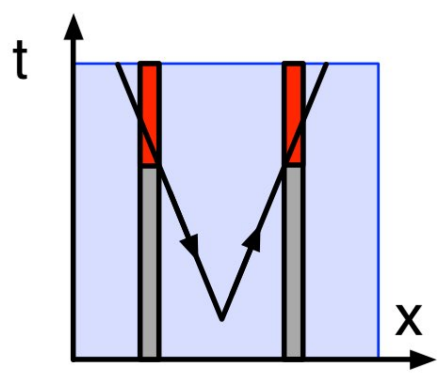

FIG. 12 – We perform an experiment to test the EPR paradox. We create a particle/antiparticle pair, which necessarily have opposite spins. We then use machines to measure the spins of the particles.

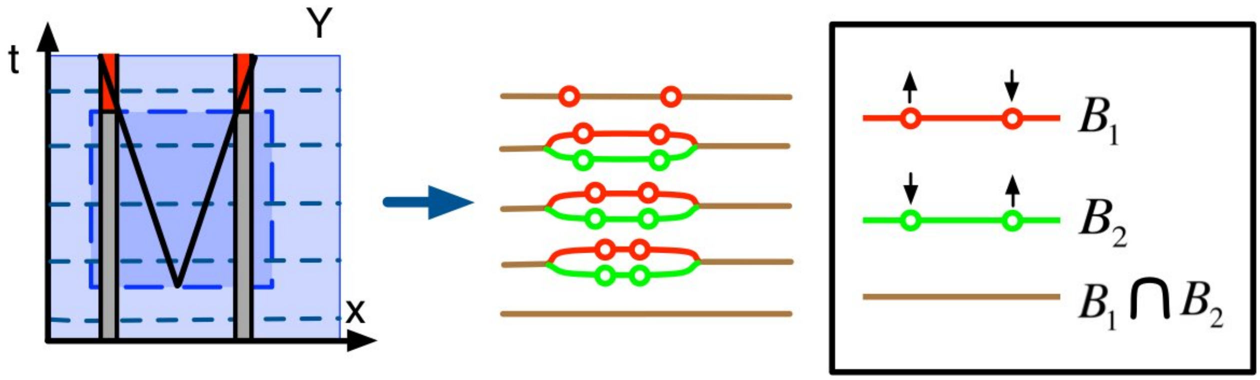

FIG. 13 – In this figure, we show the simplest version of entanglement on a branched 1+1-manifold. We produce a particle/antiparticle pair and show time slices of the manifold. In the first slice, the branches are not separated and there are no particles. In the next three slices, there is a particle/antiparticle pair and the two branches are separated. There is a branch state labeled in red, for which the left particle has spin up and the right particle has spin down. There is a branch state labeled in green, for which the left particle has spin down and the right particle has spin up. The entanglement information is encoded in the way that the knot spins are associated to their branches. When the states are measured, the branches recombine and the knots collapse to just one spin state, in this case red. (For simplicity, the machine is not shown on the branch slices. It is implicitly there, forcing the knots to collapse to one state or the other.)

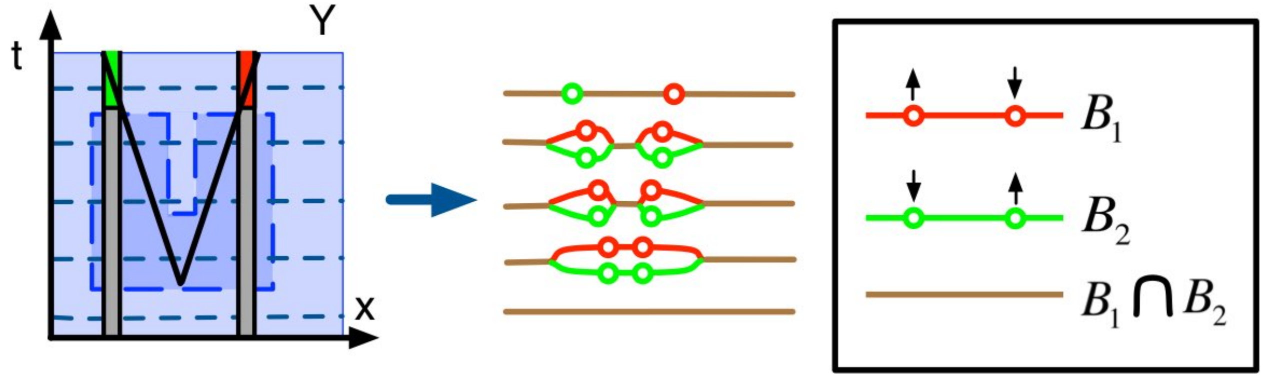

FIG. 14 – In this figure we again see the production of a particle/antiparticle pair. In the first slice, the branches are not separated and there are no particles. In the second slice, the branches have separated and there is a particle/antiparticle pair. On each of the branches, the particle knot has opposite spin to the antiparticle knot. In the third slice, the branches have recombined in the middle, which causes a loss of the entanglement. In the third and fourth slices, the particles are in a product state. The collapse of state of the two particles is independent and this leads to the possibility of a green state on the left and a red state on the right, which is shown in the last slice. This is an example of loss of entanglement. In the remainder of this paper, we will examine factors that suppress this loss of entanglement.

B. Dimension

We now increase the dimension of the manifold from 1+1 to 2+1. In Fig. 15, we see the creation of a particle/antiparticle pair, and a constant time slice on a 2+1-manifold. In Fig. 16, we see the same experiment, but we also decompose the slice to its constituent branches B_1 (labeled red) and B_2 (labeled green) that differ in the spins of the particles. In Fig. 17 we see the same experiment, but with a later time slice. After more time has elapsed, the branches B_1 and B_2 have had time to recombine with each other. The locations of their recombinations are denoted by the brown patches. In Fig. 18 we see how entanglement is preserved on the 1-manifold compared to the 2-manifold. Entanglement is preserved on the 1-manifold when there are no branch recombinations between the particles. In the case of the 2-manifold, there may be many branch recombinations that still do not suffice to separate the quantum states. In Fig. 19, we see how to separate the quantum states of the two particles for the 1-dimensional case and the 2-dimensional case. For the 1-dimensional case, any recombination between the particles separates their quantum states. For the 2-dimensional case, we see a recombination $B_1 \cap B_2$ that completely surrounds the right particle. For that reason, there exists a single branch that has the left particle in state B_1 and the right particle in state B_2. In general, to separate the particle states there must be a "cut" that separates one particle from the other and which lies entirely within a branch intersection. If this happens, then the entanglement has been lost. For the 1-dimensional case, this condition is relatively common. For the 2-dimensional case, the condition is harder to achieve, and it is even more difficult in the 3-dimensional case.

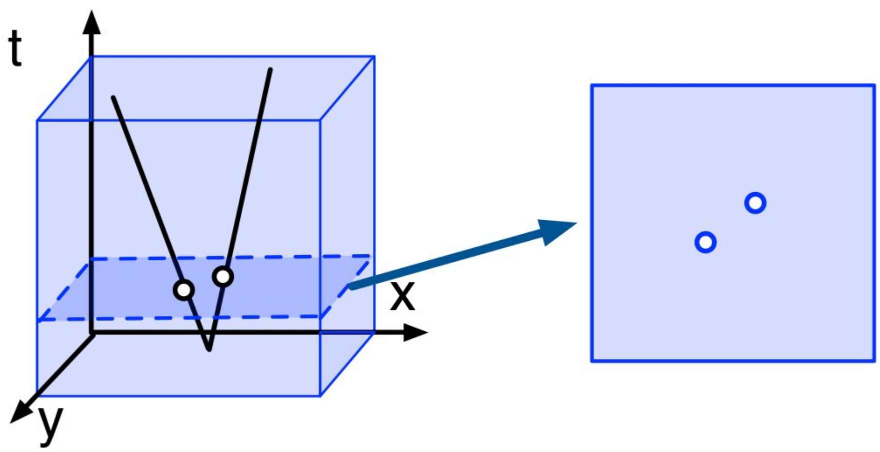

FIG. 15 – This figure shows the EPR experiment performed on a 2+1-manifold. The particle/antiparticle pair is created and separated. We take a time slice and show the location of each of the particles in that time slice.

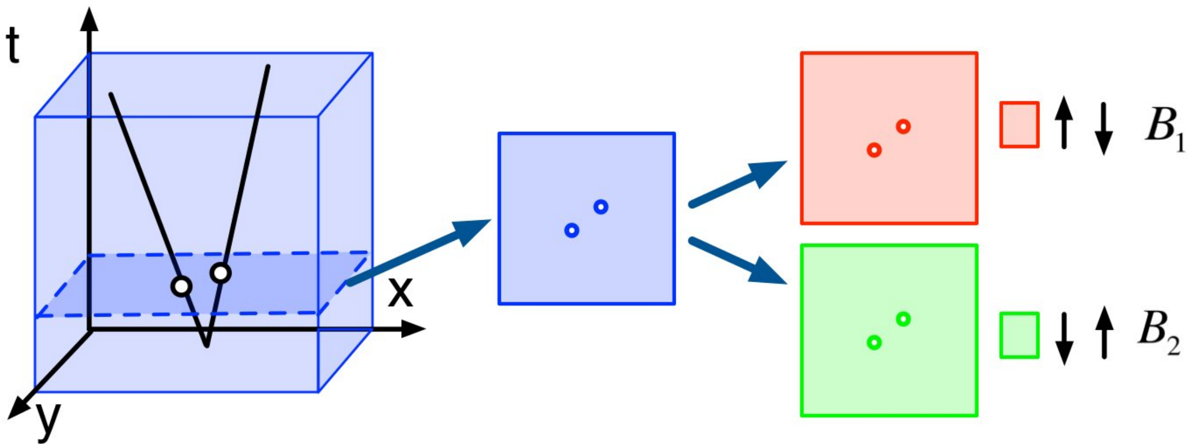

FIG. 16 – This figure shows the EPR experiment on a branched 2+1-manifold. We decompose the time slice and find that it consists of two branches (in this example). The branch B_1 (labeled red) has the left particle in spin up and the right particle in spin down. The branch B_2 (labeled green) has the left particle spin down and the right particle spin up.

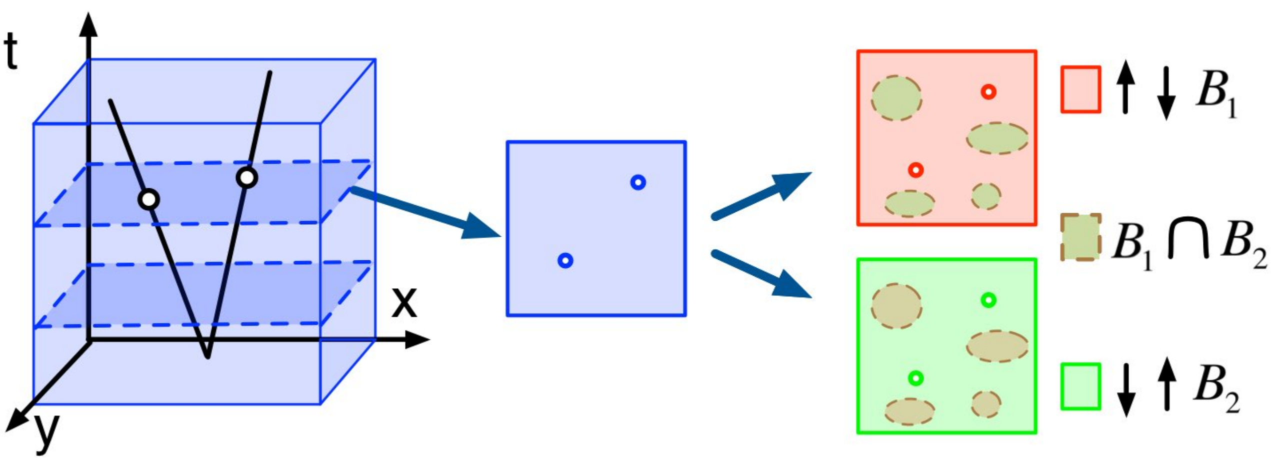

FIG. 17 – We use the same experiment as the one depicted in Fig. 16, but we now look at a later time slice. In the later slice, we see that recombinations have occurred between branches B_1 and B_2. On the branches, the location of those recombinations ($B_1 \cap B_2$) is labeled in brown.

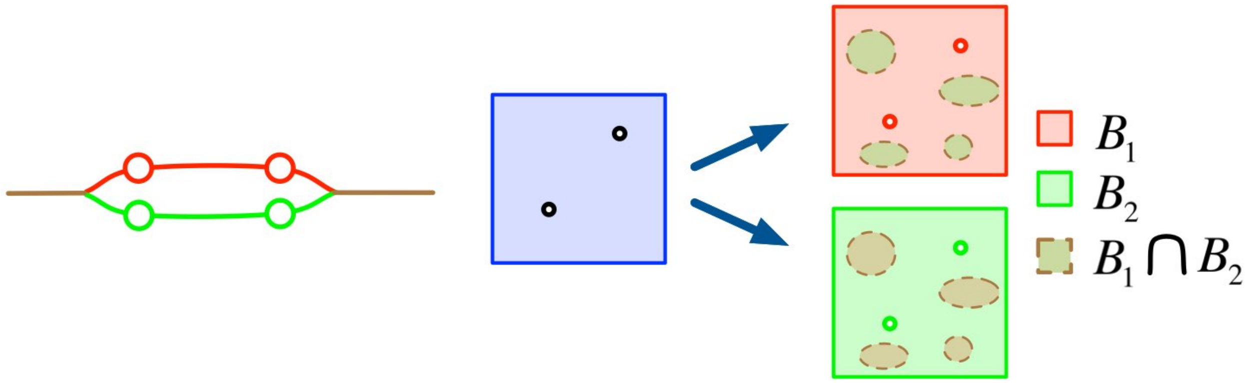

FIG. 18 – In this figure we show entanglement being preserved on a 1+1-manifold and a 2+1-manifold. In the case of the 1+1-manifold, entanglement is preserved when there are no recombinations between the particles. In the case of the 2+1-manifold, there can be many branch recombinations that still do not break the entanglement between the left particle and the right particle.

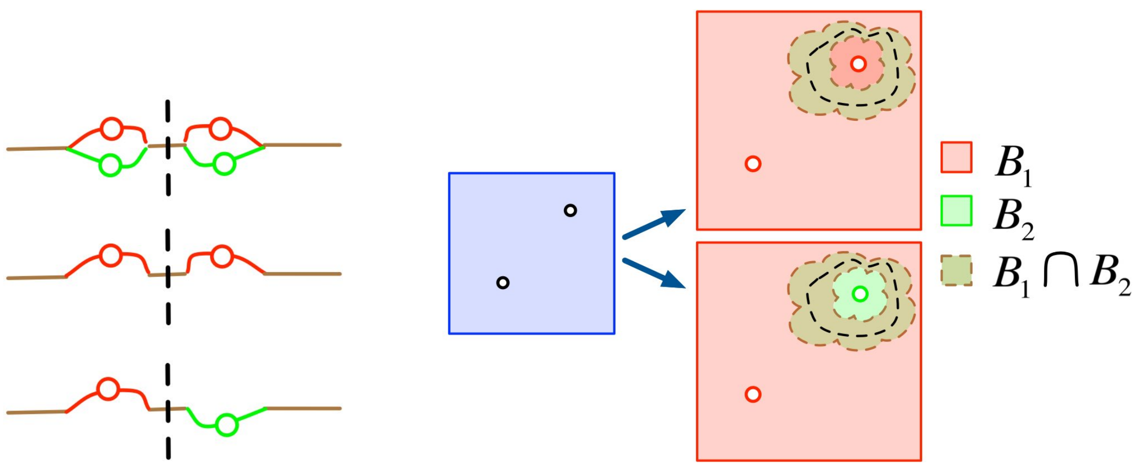

FIG. 19 – In this figure we show entanglement being lost on a 1+1-manifold and a 2+1-manifold. In both cases, the dotted line shows a separation between the left particle and the right particle. We see how the red state of the left particle can be on a single branch that contains the red state on the right, but can also be on a single branch that contains the green state on the right. This indicates a loss of entanglement. We note that loss of entanglement is less likely on the 2+1-manifold.

C. Number of states

Bell's inequality results from the measurement of the spins of particles, and our previous description has focused on the case that particles can have one of two spins. Actual particles have a continuum of possible spins and the knots only recombine with each other if the spins are similar (or the recombination is forced by collapse of state). Effectively, particle spins have a large number of distinct states. This large number of states is also a reason that entanglement will tend to be preserved. In Fig. 20, we see the B_1 branch of a time slice of a 2+1-manifold, and it has many intersections with other branches. The other branches all have distinct particle spins. We attempt a cut that separates the left particle from the right particle, but that cut does not suffice to connect two different quantum states of the left particle and the right particle. In the figure, we see that the cut is on the intersections with B_2, B_3, and B_4. A branch that is on both sides of that cut path must be in $(B_1 \cup B_2) \cap (B_1 \cup B_3) \cap (B_1 \cup B_4)$, which means that the only branch that can be on both sides of the cut is B_1, which implies that the entanglement is preserved. As the number of distinct states increases, the probability of preserving entanglement increases.

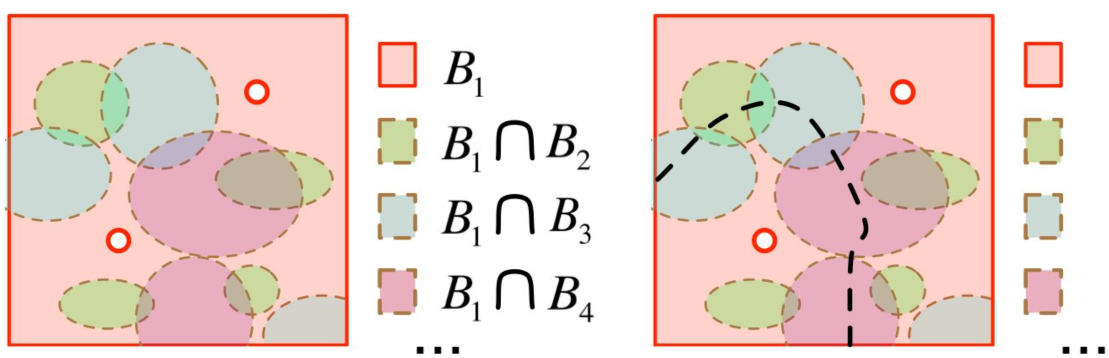

FIG. 20 – In the left diargam we see the branch B_1 and its intersections with branches B_2, B_3, B_4, etc. There are more than two spin states for a particle, and there are more than two possible branches. In the right diagram we see an attempted cut to separate the left particle from the right particle. Given this cut, if the left particle is on branch B_1, then the right particle must be on $(B_1 \cup B_2) \cap (B_1 \cup B_3) \cap (B_1 \cup B_4)$, which means that it is on B_1. This cut therefore does not imply a loss of entanglement between the left particle and the right particle.

D. Spacelike separation

We now return to the EPR experiment to examine how it appears on the branched manifold. In Fig. 21 we see the apparatus of the experiment and a constant time slice. We look at that constant time slice and see that the measurement of the left particle can only be in a different state from the measurement of the right particle if there exists some cut that separates them. We also note that there is no conflict from performing these measurements at spacelike physical separation. The collapse of state associated with measurement is a consequence of the recombination of branches of the manifold, and we are reminded in Fig. 22 that branch recombination is not constrained to occur along causal boundaries.

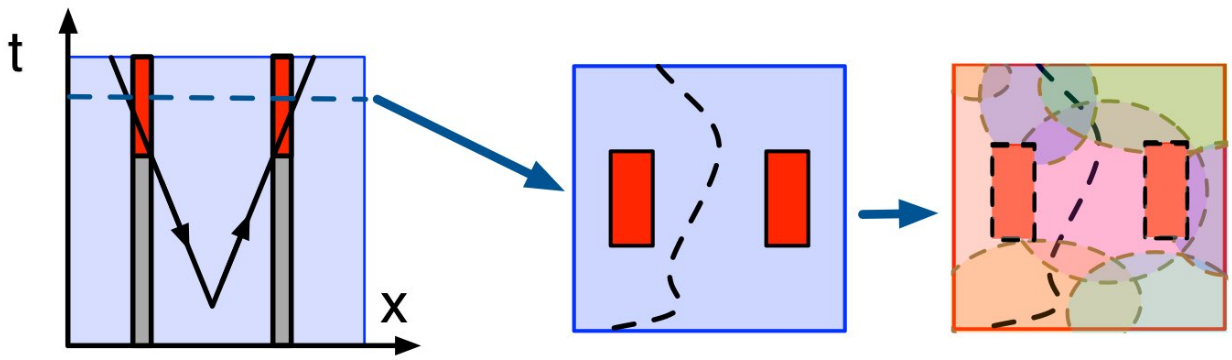

FIG. 21 – We can now show what happens in the EPR experiment on a branched manifold. The particle/antiparticle pair is created, physically separated, and measured. We take a slice of the manifold at a time shortly after measurement. Measurement forces a collapse of state based on the spins of the particles. We can attempt a cut that separates the left particle from the right particle, but if no cut suffices then the state of both of them will collapse to the same state, in this case labeled red. In this paper, we have shown how the dimension of the manifold and number of possible branch states both contribute to preserving entanglement.

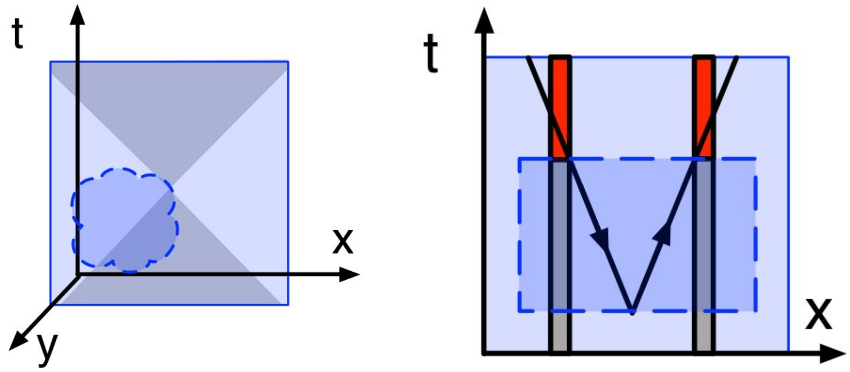

FIG. 22 – There is no requirement that the boundaries of branch separation stay within causal cones. The left diagram shows a branch separation and its relationship to a causal cone. Because the boundary of branch separation can be outside the causal cone, the collapse of state associated with entangled particles can also occur outside the causal cone. For example, in the EPR experiment, measurement of the pair of particles can be correlated even when they are at spacelike physical separation.

IV. CONCLUSION

In this paper we showed how a branched spacetime manifold can maintain quantum information and how that quantum information can affect measurements, even if the measurements occur at spacelike separation. The EPR paradox thought experiment shows that entanglement is a necessary component of quantum theory, though quantum mechanics does not describe physical features that contain the information of that entanglement. In knot physics, quantum mechanics is a consequence of the interactions of the branches of the spacetime manifold, and the information of entanglement is encoded in the branches of the manifold. A particle in this theory corresponds to knots on the branches of the manifold. One real particle has one knot on every branch. The probability distribution of quantum states for an individual particle is equal to the probability distribution of the knots on the branches. For multiple particles, the probability distributions of the corresponding knots on the branches may be correlated. In this case, we say that the particles are entangled. We showed that the geometry of the branched manifold has a tendency to preserve the entanglement of particles, even if they are physically separated from each other. In knot physics, collapse of state occurs naturally as a consequence of branch recombination. The recombination of branches occurs along branch separation boundaries that can be outside of the causal cone. For that reason, measurement of entangled particles can have correlated results even when the measurements are performed at spacelike separation.

V. APPENDIX

Knot physics is a unification theory that assumes spacetime is a branched manifold embedded in a Minkowski space. The theory is described in [1], and that description will be helpful for a more complete understanding of this paper. We list the assumptions of knot physics here for reference.

- **We assume a Minkowski 6-space Ω.** The metric on Ω is $\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1, -1, -1)$. The coordinates are x^ν.
- **We assume a branched 4-manifold M embedded in Ω.** A *branch* of M is any closed unbranched 4-manifold B without boundary that is contained in M. The metric $\bar{\eta}_{\mu\nu}$ on M is inherited from Ω. For convenience of coordinates we assume that, if M is flat, then M is in the subspace spanned by x^0, x^1, x^2, x^3.
- **We assume non-self-intersection of each branch of M.** For any branch B, the branch B cannot intersect itself. This is necessary to prevent knots from spontaneously untying.
- **We assume a vector field A^ν.** The field satisfies $\det(A_{\alpha,\mu}A^\alpha{}_{,\nu}) = -1$.
- **We assume a conformal weight ρ.** Then we define the metric $g_{\mu\nu} = \rho^2 A_{\alpha,\mu}A^\alpha{}_{,\nu}$ and a Ricci curvature $\hat{R}^{\mu\nu}$ based on $g_{\mu\nu}$.
- **We assume a constraint on $g_{\mu\nu}$ relative to $\eta_{\mu\nu}$.** The metrics $g_{\mu\nu}$ and $\eta_{\mu\nu}$ define sets g^+ and η^+, and we assume that g^+ must intersect η^+.
- **We assume Ricci flatness $\hat{R}^{\mu\nu} = 0$ for $g_{\mu\nu}$.**
- **We assume that the weight $w = (-\det(g))^{1/2} = \rho^4$ is conserved at branching.**
- **We assume a lower limit $w \geq 1$.** This implies that the manifold can branch only a finite number of times.

[1] C. Ellgen **www.knotphysics.net** Physics on a Branched Knotted Spacetime Manifold