Knot Physics: Dark Energy

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Abstract

We assume that spacetime is embedded in a Minkowski space and the metric on spacetime is induced by the Minkowski metric. Expansion of spacetime causes a redshift that corresponds to the usual cosmological redshift of general relativity. Changing expansion velocity also affects the redshift and introduces an additional term that is not included in the redshift effect attributed to general relativity. This extra contribution may explain the difference between astronomical data and the redshift predictions of general relativity.

We want to determine the redshift of photons on a spacetime manifold that is embedded in a Minkowski space and has the induced metric. We find that some of the redshift is due to the expansion of the spacetime manifold and this redshift is equivalent to the redshift in general relativity. There is, however, an additional contribution to the redshift that results from the changing velocity of the expansion rate. This extra contribution may explain the difference between astronomical data and the redshift predictions of general relativity.

We will use a toy model such that the Minkowski space is 2+1 with polar coordinates (t, ρ, θ) and the spacetime manifold is an embedded 1+1 manifold. Because photons travel 1-dimensional paths, this model will be adequate to describe a photon's path and its redshift. Let each constant t slice of spacetime be a circle centered at the origin with radius $r(t)$. Then the manifold is embedded as $(t, r(t), \theta)$. The cosmology of this universe corresponds to change of the radius r. An expanding universe has an increasing radius, $\dot{r} > 0$.

Because the speed of light is 1, a photon's wavelength λ is equal to the time delay between the front edge of the photon and its back edge. At the time of departure, the photon has wavelength λ_1 and at the time of arrival the photon has wavelength λ_2. The front edge of a photon travels a path $p_f(t)$ that departs at $(t_1, r(t_1), \theta_1)$ and arrives at $(t_2, r(t_2), \theta_2)$. The back edge of the photon travels a path $p_b(t)$ that departs at $(t_1 + \lambda_1, r(t_1 + \lambda_1), \theta_1)$ and arrives at $(t_2 + \lambda_2, r(t_2 + \lambda_2), \theta_2)$.

For a path $p(t) = (t, r(t), \phi(t))$ the arc length is

$$L(p) = \int \sqrt{\left(r(t)\dot{\phi}(t)\right)^2 + \dot{r}(t)^2}\ \mathrm{d}t \tag{1}$$

The difference of the path lengths for front and back edges of the photon is $L(p_b) - L(p_f)$. This difference is also equal to the change in wavelength, $L(p_b) - L(p_f) = \lambda_2 - \lambda_1$.

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Using the fact that the speed of the photon is always 1, we have

$$\big(r(t)\dot{\phi}(t)\big)^2 + \dot{r}(t)^2 = 1 \tag{2}$$

Some of the velocity of the photon is in the angular direction and corresponds to $r(t)\dot{\phi}(t)$. Some of the velocity of the photon is in the radial direction and corresponds to $\dot{r}(t)$. Because the speed is always 1, if more of the photon velocity is in the radial direction then less of the velocity is in the angular direction and this implies that it takes the photon more time to proceed from angular coordinate θ_1 to θ_2. A changing radial velocity $\dot{r}(t)$ may cause the back edge of the photon to require a different amount of time to travel from θ_1 to θ_2 compared to the front edge. The back edge is delayed from the front edge by λ_1 at departure and λ_2 at arrival. We can solve for the redshift, which is the ratio λ_2/λ_1.

$$\dot{\phi}(t) = \frac{\sqrt{1-\dot{r}(t)^2}}{r(t)} \tag{3}$$

$$\int_{t_1}^{t_2} \dot{\phi}(t)\ \mathrm{d}t = \theta_2 - \theta_1 = \int_{t_1+\lambda_1}^{t_2+\lambda_2} \dot{\phi}(t)\ \mathrm{d}t \tag{4}$$

$$\int_{t_1+\lambda_1}^{t_2+\lambda_2} \dot{\phi}(t)\ \mathrm{d}t - \int_{t_1}^{t_2} \dot{\phi}(t)\ \mathrm{d}t = 0 \tag{5}$$

We say that $\dot{\phi}(t)$ is approximately constant in the interval between t_1 and $t_1 + \lambda_1$ and also approximately constant in the interval between t_2 and $t_2 + \lambda_2$. Then we can approximate

$$\int_{t_1+\lambda_1}^{t_2+\lambda_2} \dot{\phi}(t)\ \mathrm{d}t - \int_{t_1}^{t_2} \dot{\phi}(t)\ \mathrm{d}t \approx \lambda_2\dot{\phi}(t_2) - \lambda_1\dot{\phi}(t_1) = 0 \tag{6}$$

From this we derive that the redshift is the ratio of wavelengths

$$\frac{\lambda_2}{\lambda_1} \approx \frac{\dot{\phi}(t_1)}{\dot{\phi}(t_2)} = \frac{r(t_2)\sqrt{1-\dot{r}(t_1)^2}}{r(t_1)\sqrt{1-\dot{r}(t_2)^2}} \tag{7}$$

The redshift of general relativity is

$$\frac{\lambda_2'}{\lambda_1'} = \frac{r(t_2)}{r(t_1)} \tag{8}$$

and the additional factor of equation (7) may be useful for distinguishing between cosmological models in which the universe is either embedded in a larger space or not.