Knot physics: neutrino helicity

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Abstract

We use the assumptions of knot physics to prove that a collection of interacting neutrinos and antineutrinos maximize their quantum probability when all neutrinos are of the same helicity and all antineutrinos are of the opposite helicity. Knot physics demonstrates that the geometry of gravity spontaneously breaks symmetry. We show here that the geometry of gravity couples the neutrino linear momentum to its quantum phase. Likewise, the quantum phase of an interacting neutrino couples to its spin angular momentum. Therefore, the symmetry breaking of gravity couples the linear momentum of an interacting neutrino to its spin angular momentum, producing consistent helicity.

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I. INTRODUCTION

This paper will use many of the assumptions from the paper "Knot physics, spacetime in co-dimension 2" [1] (available at **www.knotphysics.net**), which is necessary background reading. In particular, we show in that paper that gravity spontaneously breaks symmetry. We show in this paper how that spontaneous symmetry breaking produces the symmetry breaking of neutrino helicity.

II. NEUTRINO HELICITY

For a collection of interacting neutrinos and antineutrinos, the quantum probability is optimized when the neutrinos all have the same helicity and the antineutrinos all have the opposite helicity. To show this, we first show that gravity couples the neutrino's linear momentum to its quantum phase. Then we show that neutrino interactions couple the spin angular momentum of the neutrino to its quantum phase. Because of those couplings, a fixed relationship between the neutrino linear momentum and spin angular momentum optimizes the quantum probability of a collection of interacting neutrinos.

A. Gravity couples linear momentum to quantum phase

Locally, express the points on M as $(x_0, x_1, x_2, x_3, bsin(y), bcos(y))$ for some variables b and y. We will use y to show the relation between gravitational rotation, spin angular momentum, and quantum phase.

Gravity breaks parity. If $A^\nu = x^\nu$ then parity is broken by rotation of the form $(x_0, x_1, x_2, x_3, bsin(k^\nu x_\nu), bcos(k^\nu x_\nu))$ for a causal vector field k^ν. However, A^ν constrains the causality of points on M. Therefore, for a general A^ν field, the gravitational rotation is $(x_0, x_1, x_2, x_3, bsin(k^\nu A_\nu), bcos(k^\nu A_\nu))$, which means $y = k^\nu A_\nu$ where there is no particle.

We describe a $S^1 \times P^2$ with a mapping:

$$(r, \theta, \phi, 0, 0) \to (g(r), \theta, \phi, h(r)sin(2\theta + \omega t), h(r)cos(2\theta + \omega t)) \quad (1)$$

which means that $y = 2\theta + \omega t$ close to the particle, ignoring the gravitational background. The $S^1 \times P^2$ can also have opposite θ orientation:

$$(r,\theta,\phi,0,0) \to (g(r),\theta,\phi,h(r)sin(-2\theta+\omega t),h(r)cos(-2\theta+\omega t)) \quad (2)$$

which means that $y=-2\theta+\omega t$ close to the particle, ignoring the gravitational background.

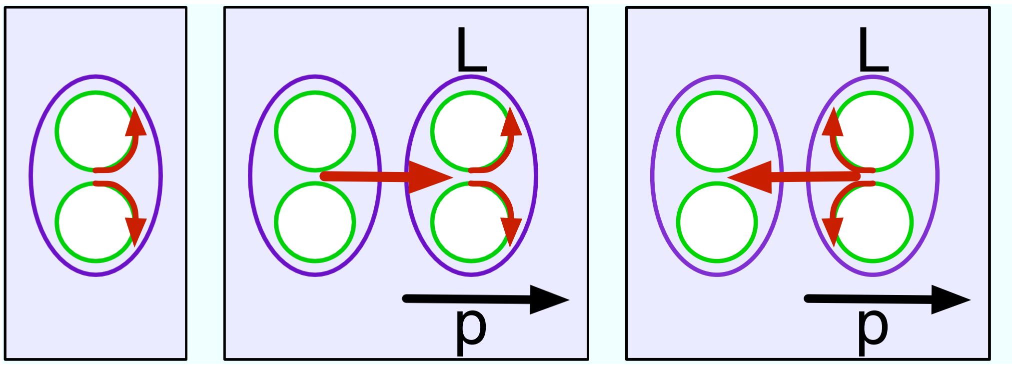

FIG. 1: The diagrams show constant ϕ slices of $S^1 \times P^2$. The red arrows are $\partial^\mu y$. The left diagram shows $\partial^\mu y$ on a $S^1 \times P^2$, in the $+\theta$ direction. The other two diagrams show a lepton L becoming a neutrino. The lepton transfers charge to another $S^1 \times P^2$. The transfer of charge gives L momentum p in the opposite direction. Gravity produces a background rotation, with $\partial^\mu y = \partial^\mu(k_\nu A^\nu)$. Because of the charges on each $S^1 \times P^2$, $\partial^\mu(k_\nu A^\nu)$ points either towards L (on the left) or away from L (on the right). The vector field $\partial^\mu y$ must be consistent, therefore $\partial^\mu(k_\nu A^\nu)$ must match the $+\theta$ or $-\theta$ arrows on L. The sign of the charge determines $\partial^\mu(k_\nu A^\nu)$. Therefore the relation between quantum phase θ and the linear momentum p for neutrinos is opposite to that of antineutrinos. Beginning with a neutrino L and producing a charged lepton would reverse the direction of p in the diagram.

Every neutrino is created when a charged lepton L annihilates its charge with the charge of another particle, see Fig. 1. At creation, $y = k_\nu A^\nu$ must be consistent with $y=+2\theta+\omega t$ or $y=-2\theta+\omega t$. At the charge annihilation, the $A^{0,\mu}$ field is lightlike, which determines whether $\partial^\mu y = \partial^\mu(k_\nu A^\nu)$ points towards the neutrino or away from it. Therefore, the sign of the charges determines whether the $S^1 \times P^2$ orientation is $+\theta$ or $-\theta$. The linear momentum of the neutrino is equal and opposite to the momentum of the charge that leaves the neutrino. Therefore the transfer of charge determines the relationship between the neutrino's linear momentum and quantum phase. The charge that produces a neutrino is opposite to that of an antineutrino, therefore the relationship between linear momentum and quantum phase is also opposite.

B. Neutrino interaction couples spin angular momentum to quantum phase

Neutrinos have a large S^1 radius (see [1]) of about $10^{-5}m$, are abundant, and travel at relativistic velocities. The neutrino/neutrino interactions are frequent. Fig. 2 shows a pair of neutrinos interacting such that one neutrino passes through the center of the other. If two particles are adjacent to each other such that each P^2 slice of one is adjacent to a P^2 slice of the other, then the quantum phases of the two interact. Describe the quantum phase of particle a and particle b using a complex number to describe the x_4 and x_5 coordinates. Then the particle amplitudes are $h_a exp(iy_a)$ and $h_b exp(iy_b)$ and their interaction produces $(1/2)(h_a exp(iy_a) + h_b exp(iy_b))$, which affects the quantum phase amplitude. The derivatives $\partial^\mu y_a$ and $\partial^\mu y_b$ also affect the quantum phase. Take a spacelike slice of a pair of interacting $S^1 \times P^2$ and further restrict the slice to constant ϕ. Let $\Pi_\phi(x)$ be the projection of a vector x onto that slice. If the derivatives satisfy $\Pi_\phi(\partial^\mu y_a) = -\Pi_\phi(\partial^\mu y_b)$ then the sum $(1/2)(h_a exp(iy_a) + h_b exp(iy_b))$ collapses the geometry of both P^2 in the slice, which reduces the probability. For fixed y_a and y_b, the derivatives that optimize the quantum probability are the ones such that $\Pi_\phi(\partial^\mu y_a) = \Pi_\phi(\partial^\mu y_b)$. When one neutrino passes through another, their quantum phases interact as in Fig. 3. If both are neutrinos or both are antineutrinos then they have $\Pi_\phi(\partial^\mu y_a) = \Pi_\phi(\partial^\mu y_b)$. If one is a neutrino and the other is an antineutrino then they have $\Pi_\phi(\partial^\mu y_a) = -\Pi_\phi(\partial^\mu y_b)$. However, the spin angular momenta can also contribute to $\partial^\mu y$. The spin angular momentum is random ripples circulating along the S^1 fiber. We represent the random ripples as a random function $s(\phi + vt)$.

$$(r, \theta, \phi, 0, 0) \rightarrow (g(r), \theta, \phi, h(r)sin(\pm 2\theta + s(\phi + vt) + \omega t), h(r)cos(\pm 2\theta + s(\phi + vt) + \omega t)) \quad (3)$$

Then $y = \pm 2\theta + s(\phi + vt) + \omega t$ and we will write the gradient $\partial^\mu y$ as $y' = \pm 2\hat{\theta} + (\hat{\phi} + v\hat{t})s' + \omega\hat{t}$ where $\hat{\theta}$, $\hat{\phi}$, and $\hat{t}$ are the unit vectors along those axes and s' is the derivative of the s function. For a relativistic neutrino, the geometry Lorentz transforms. We can consider the quantum interaction of two neutrinos in the frame where the neutrinos have equal but opposite velocity. In that frame the rotation rates of their spin angular momenta are equal. Neutrino a and neutrino b have velocities $\beta_a = -\beta_b$. Let c_a and c_b determine the sign of θ where their quantum phases interact, for example $y'_a = c_a 2\hat{\theta} + (\hat{\phi} + v_a\hat{t})s'_a + \omega\hat{t}$. There is either $c_a = c_b$ (if both are neutrinos or antineutrinos) or $c_a = -c_b$ (if one is a neutrino and one is an antineutrino). The value of ω is the same for both because they both couple to the background gravitational rotation. The functions s'_a and s'_b are independent and random. The spin velocities v_a and v_b are constants that determine the spin direction and have the same magnitude $|v_a| = |v_b|$. To consider the interaction of a pair of relativistic neutrinos we use the Lorentz transformation of y' on the inner and outer edge of the neutrino, which is where they interact. On those edges, we have the Lorentz transformations from the vectors $\hat{t}_r$, $\hat{\theta}_r$, and $\hat{\phi}_r$ in the rest frame:

$$\hat{t}_r \to \gamma\hat{t} - \beta\gamma\hat{\theta} \tag{4}$$

$$\hat{\theta}_r \to -\beta\gamma\hat{t} + \gamma\hat{\theta} \tag{5}$$

$$\hat{\phi}_r \to \hat{\phi} \tag{6}$$

Therefore in the frame where the neutrinos have velocities β_a and β_b.

$$y'_a = c_a 2(-\beta_a\gamma\hat{t} + \gamma\hat{\theta}) + (\hat{\phi} + v_a\gamma\hat{t} - \beta_a v_a\gamma\hat{\theta})s'_a + \omega\gamma\hat{t} - \omega\beta_a\gamma\hat{\theta} \tag{7}$$

$$y'_b = c_b 2(-\beta_b\gamma\hat{t} + \gamma\hat{\theta}) + (\hat{\phi} + v_b\gamma\hat{t} - \beta_b v_b\gamma\hat{\theta})s'_b + \omega\gamma\hat{t} - \omega\beta_b\gamma\hat{\theta} \tag{8}$$

In the interaction frame, the geometry is determined by the spacelike components. Let the projection $Space(x)$ be the projection such that $Space(\hat{t}) = 0$. Then

$$Space(y'_a) = \gamma(2c_a - \omega\beta_a)\hat{\theta} + (\hat{\phi} - \beta_a v_a\gamma\hat{\theta})s'_a \tag{9}$$

$$Space(y'_b) = \gamma(2c_b - \omega\beta_b)\hat{\theta} + (\hat{\phi} - \beta_b v_b\gamma\hat{\theta})s'_b \tag{10}$$

Consider the interaction of neutrino a and neutrino b halfway between the inner edge of a and the outer edge of b. Because s'_a and s'_b are random functions, the vectors $(\hat{\phi} - \beta_a v_a\gamma\hat{\theta})s'_a$ and $(\hat{\phi} - \beta_b v_b\gamma\hat{\theta})s'_b$ have random magnitude. We want to find the spin velocities v_a and v_b that statistically optimize the interaction of y'_a and y'_b.

The quantum probability of each neutrino can be determined by taking the amplitude of constant $\phi + vt$ slices through the particle in the rest frame. In the interaction frame, these are the slices perpendicular to $\hat{\phi} - \beta v\gamma\hat{\theta}$. Call the projections of vectors onto those slices $\Pi_a(x)$ and $\Pi_b(x)$ such that $\Pi_a(\hat{\phi} - \beta_a v_a\gamma\hat{\theta}) = 0$ and $\Pi_b(\hat{\phi} - \beta_a v_a\gamma\hat{\theta}) = 0$, each slice eliminates the random contributions from the spin angular momentum of one neutrino. The slices Π_a and Π_b are the same slice if $v_a\beta_a = v_b\beta_b$, in which case the random contributions from both neutrinos are zero in that slice.

If $c_a = c_b$ then both $Space(y'_a)$ and $Space(y'_b)$ share a common term $2\gamma\hat{\theta}$. We can optimize the probability by removing the random spin angular momentum terms in $\Pi_a(y'_b)$ and $\Pi_b(y'_a)$.

(The quantum probability is of the form $P = exp(\int_0^{2\pi} ln(r(\phi)^2)d\phi)$ and ln is a convex function, therefore random contributions to $r(\phi)$ reduce the expected value of the integral). To remove the random spin terms from $\Pi_a(y'_b)$ and $\Pi_b(y'_a)$ we choose $v_a\beta_a = v_b\beta_b$ which implies $v_a = -v_b$. Therefore the spin directions are opposite in the interaction frame and the helicities of the neutrinos are the same.

If $c_a = -c_b$ then $\gamma(2c_a - \omega\beta_a)\hat{\theta} = -\gamma(2c_b - \omega\beta_b)\hat{\theta}$, the non-random terms are exactly opposite. We therefore optimize the probability by reducing the chance that the random terms are also exactly opposite, $(\hat{\phi} - \beta_a v_a \gamma\hat{\theta})s'_a = -(\hat{\phi} - \beta_b v_b \gamma\hat{\theta})s'_b$. If the random components are parallel then they are opposite if $s'_a = -s_b$. If they are not parallel then they are opposite only if $s'_a = s'_b = 0$. We therefore conclude that the random components optimize probability if they are not parallel. This implies that $v_a\beta_a = -v_b\beta_b$, which implies that $v_a = v_b$. Therefore their spin directions are the same in the interaction frame and their helicities are opposite.

To summarize, for neutrino/neutrino and antineutrino/antineutrino interactions we have $c_a = c_b$ and probability is optimized when the helicities are the same. For neutrino/antineutrino interactions we have $c_a = -c_b$ and probability is optimized when helicities are opposite, see Fig. 4.

C. Baryon asymmetry

The distinction between neutrinos and antineutrinos is the consequence of two spontaneously broken symmetries. The first symmetry is the parity breaking of the gravitational background rotation. The second symmetry is the spin angular momenta of neutrinos and antineutrinos. If these symmetries were previously unbroken then the production of neutrinos and antineutrinos would have had random quantum phase and spin angular momenta. After symmetry breaking, there would be some number of neutrinos and antineutrinos but no reason to assume that those numbers would be exactly equal. It is reasonable to expect that one type would have a slight excess that would lead to excess matter in the universe.

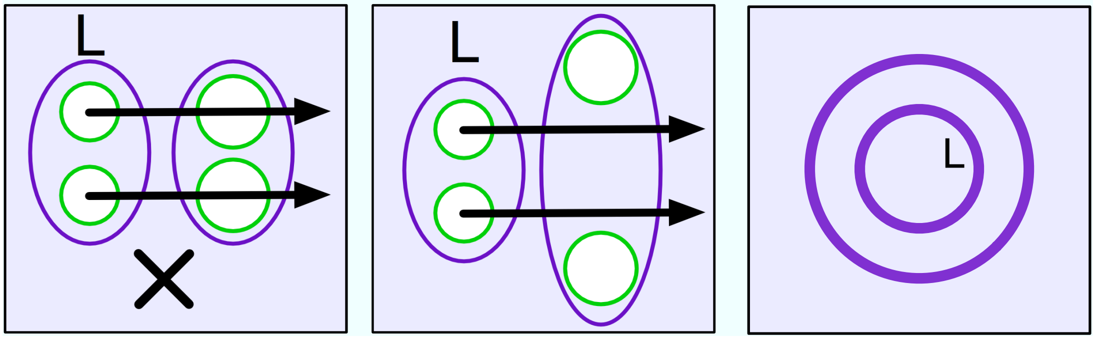

FIG. 2: The left diagram shows a constant ϕ slice of a neutrino L passing through another neutrino with each P^2 slice of L passing through a P^2 slice. Topologically, this is allowed. However, it reverses the θ orientation of L. The quantum branches such that L reverses its θ orientation recombine with the branches where L does not reverse θ orientation. The recombination has geometry that collapses the $S^1 \times P^2$ and has zero probability. Therefore this type of pass-through is not allowed. The middle diagram shows a constant ϕ slice of an allowed pass-through. The right diagram shows the same pass-through in a constant θ slice. Because of the large S^1 radius of neutrinos (see [1]) and the abundance of neutrinos, these pass-throughs are common.

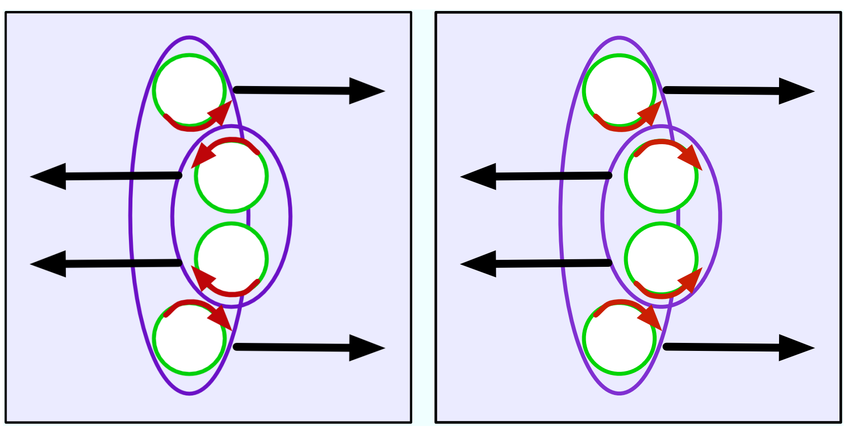

FIG. 3: The diagrams show constant ϕ slices of one neutrino passing through another neutrino. The red arrows are $\partial^\mu y$. As above, they indicate the θ orientation of the P^2. The quantum phase amplitude is maximized when the red arrows point in the same direction at the point where the P^2 slices are closest. From the previous section, we know that happens when both are neutrinos or both are antineutrinos.

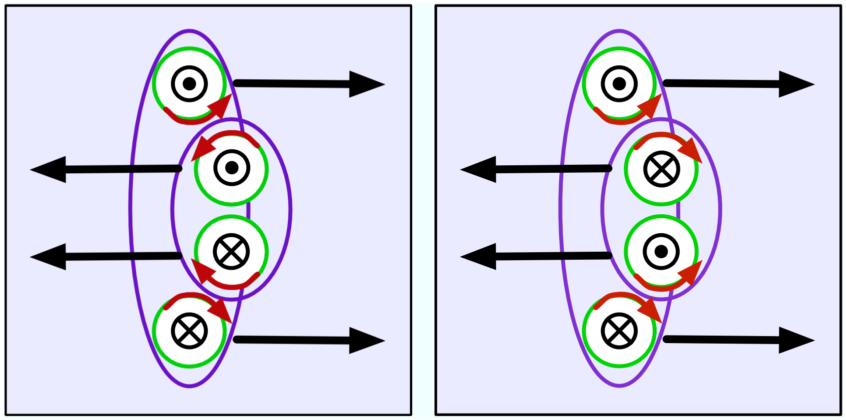

FIG. 4: The diagrams show constant ϕ slices of one neutrino passing through another neutrino with vectors indicating their spin angular momenta. The red arrows are $\partial^\mu y$. The spin angular momenta are those which make the red arrows $\partial^\mu y$ as consistent as possible after Lorentz transformation. On the left is a neutrino/antineutrino interaction; the spin angular momenta reduce the opposition of the $\partial^\mu y$. On the right is a neutrino/neutrino interaction; the spin angular momenta preserve the alignment of the $\partial^\mu y$ after Lorentz transformation. The helicity for neutrinos is consistent. The helicity for antineutrinos is consistent. The helicity of neutrinos is opposite to the helicity of antineutrinos.

[1] C. Ellgen **www.knotphysics.net** Knot physics, spacetime in co-dimension 2