

# **Knot physics: the fine structure constant. An open question.**

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## **Abstract**

This paper is being released to coincide with a conference presentation on knot physics. However, this paper is more a collection of ideas and observations than a complete work. Using the assumptions of knot physics one can derive the energy of an electron. The energy calculated is necessarily specified in the units of the calculation. Without a prior assumption of those units, the number produced cannot be compared to actual particle energy. If Planck's constant can also be expressed in the same units, then perhaps the particle energy can be compared to Planck's constant to derive the fine structure constant.

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## I. BACKGROUND

This paper will use the assumptions from the paper "Knot physics, spacetime in co-dimension 2" [1] (available at [www.knotphysics.net](http://www.knotphysics.net)), which is necessary background reading. Mathematica documents are also available at [knotphysics.net](http://knotphysics.net) that provide mathematical modeling and integration associated with the calculations here.

## II. IDEAS ABOUT THE FINE STRUCTURE CONSTANT IN KNOT PHYSICS

- To describe charge in terms of Planck's constant we need a mathematical model that describes charge, in which we can calculate Planck's constant.
- We can generate a model of an electron and then attempt to calculate Planck's constant in the units of the model.
- A model of an electron needs electron geometry, field, and  $\rho$  quantum branch weight.
- Within that model, the charge of the electron can be calculated by finding the divergence of the field using the field strength at some effectively infinite distance.
- The geometry of the electron has two degrees of freedom, the  $S^1$  radius, and the  $P^2$  radius. All other degrees of freedom are random oscillations around the basic  $S^1 \times P^2$ .
- The field of the electron has both electric and magnetic components. While the magnetic components may be important, it might be simpler to begin by considering only the electric field.
- A weak electric field has Lagrangian of the form  $F^{\mu\nu}F_{\mu\nu}$ . However, this is only an approximation. If we describe the  $A^\nu$  field as a collection of random oscillations, then the closer that  $|F^{\mu\nu}F_{\mu\nu}|$  gets to 1, the more those field oscillations are constrained. In particular, we can imagine each microscopic field oscillation as having gradient +1 or -1. For  $n$  total oscillations, with  $k$  oscillations of gradient +1, there are  $\binom{n}{k}$  possible arrangements of the microscopic oscillations. Then the Lagrangian is  $\ln\binom{n}{k}$ . If the magnitude of the overall gradient is  $A$ , we can approximate  $\ln(n!) = n \times \ln(n) - n$  to get a formula for the field energy as a function of the magnitude of the gradient  $A$ .

$$E = (1/2)[(1 - A)\ln(1 - A) + (1 + A)\ln(1 + A)] \quad (1)$$

- For a quantum branch weight  $\rho$  the density of microscopic oscillations is  $\rho$ . Therefore the field energy is

$$E = (\rho/2)[(1 - A)\ln(1 - A) + (1 + A)\ln(1 + A)] \quad (2)$$

- The  $\rho$  distribution is determined by Ricci flatness  $\hat{R}^{\mu\nu} = 0$ . For two dimensions, on a  $\mathbb{R}^2 \# P^2$ , this produces a  $\rho$  distribution based on the Gauss-Bonnet theorem:  $\int_A \hat{R} dA + \int_{\partial A} k_g ds = 2\pi\chi$ , where  $k_g$  is geodesic curvature,  $\hat{R} = 0$ , and  $\chi = 0$  is the Euler characteristic. Then  $k_g = 0$ , the circumference is constant  $C(r) = 2\pi r\rho = k$ . Therefore  $\rho$  is proportional to  $1/r$ . For a  $\mathbb{R}^3 \# (S^1 \times P^2)$ , we generate a harmonic distribution  $\kappa$  that is singular on the ring. We choose  $\kappa$  such that  $e^\kappa$  is proportional to  $1/r$  as it approaches the ring. Then  $\rho = e^\kappa$  is a Ricci flat  $\rho$  distribution in the limit that the  $P^2$  radius approaches zero. To expand the  $P^2$  radius, we make sure that  $\rho$  is conserved by the expansion. The  $\rho$  distribution is uniquely determined up to multiplication by a constant,  $\rho = \rho_0 e^\kappa$ .
- The constraint  $\hat{R}^{\mu\nu} = 0$  also constrains the electric field. The Ricci tensor is based on the metric  $g^{\mu\nu} = \rho A^{\alpha,\mu} A_{\alpha}{}^{\nu}$ . In local coordinates, Ricci flatness implies  $\partial_\beta \partial^\beta g^{\mu\nu} = 0$ . Therefore the  $A^\nu$  field has a cusp in the  $A^0$  coordinate on the circle at the center of each  $P^2$  slice of the  $S^1 \times P^2$ . Further, at the cusp  $|A^{0,\alpha}| = 1$  (using the notation from the mathematica notebooks, A=1). We generate an approximation of the rest of the Ricci-flat field by assuming that  $\partial_\mu(\rho F^{\mu\nu}) = 0$ .
- We can integrate field energy to get the total field energy of the particle. Assume a charge of  $q$  and a  $S^1$  radius of  $R$  (not the curvature). Because  $\rho$  scales with the  $P^2$  radius, the  $P^2$  radius has no effect on the field energy. To find the particle energy, we make the particle field saturated (with magnitude A=1). Then the radius of saturation is

$$R_{sat} = (2/\pi)(q/(8\pi^2))^{1/2} = (q/(2\pi^4))^{1/2} \quad (3)$$

- At saturation, the energy depends only on  $q$ . The energy is proportional to a constant term  $E_0 \approx 131.5$ .

$$E_{sat} = (q/(8\pi^2))^{3/2} E_0 \quad (4)$$

- The product  $2\pi R_{sat} E_{sat} = (q/(4\pi^2))^2 E_0$ . We note that the surface area of a torus of radii  $r_1$  and  $r_2$  is  $4\pi^2 r_1 r_2$ .
- The calculations of field energy have assumed  $\rho_0 = 1$ . In general, we have  $E_{sat} = \rho_0 (q/(8\pi^2))^{3/2} E_0$  (remembering that  $q$  also scales with  $\rho_0$ ).
- Planck's constant is known by its relation to particle energies, momenta, and uncertainty. A more general formula is probably necessary to find Planck's constant in the context of these calculations. On the branched manifold  $M$ , a 4-volume  $V$  of a branch has an action  $S(V) = \int_V L dM$ , the integral of the Lagrangian over  $V$ . Comparing the action over multiple branches, there will be an expectation value  $E[S(V)]$  and a variance  $Var[S(V)]$  for the action on the branches. If  $V$  is large enough that the action on one part of the branch can be considered uncorrelated to the other parts, then we hypothesize

$$\hbar = \frac{Var[S(V)]}{E[S(V)]} \quad (5)$$

- If the distribution of  $S(V)$  is exponential, with pdf of the form  $f(S) = \lambda e^{-\lambda S}$ , then  $Var[S] = \lambda^{-2}$  and  $E[S] = \lambda^{-1}$  and therefore  $\hbar = Var[S]/E[S] = \lambda^{-1} = E[S]$ . Then  $f(S) = \hbar e^{-\hbar S}$ .
- The product  $S_{sat} = 2\pi R_{sat} E_{sat} = (q/(4\pi^2))^2 E_0$  has the units of action. The expectation value may be related to  $\hbar$  by  $\hbar = E[S_{sat}] = 2\pi R_{sat} E_{sat} = (q/(4\pi^2))^2 E_0$ . This is a formula of the form  $\hbar = q^2 \beta$ . For fine structure constant  $\alpha$ , if  $\beta = \alpha^{-1}$  then this would be a derivation of the fine structure constant. However,  $\alpha^{-1} \approx 137$  whereas  $\beta = (1/(4\pi^2))^2 E_0$ . We note that  $E_0 \approx 131.5$  is about 4% different from the fine structure constant. The 4% difference may be due to the difference between  $A^0$  field that is actually Ricci flat and the classical approximation used in the calculation.

- If we do not assume that  $\rho_0$  must be 1, then variable  $\rho_0$  can be included in the calculation of  $S_{sat}$ , giving  $\hbar = E[S_{sat}] = 2\pi R_{sat} E_{sat} = \rho_0 (q/(4\pi^2))^2 E_0$ . It is possible that  $\rho_0$  may negate the  $(1/(4\pi^2))^2$  term.
- The constraint  $\hat{R}^{\mu\nu} = 0$  on a  $S^1 \times P^2$  is a geometric constraint that is quadratic and relates to volumes and surface areas. Therefore it may relate to the surface area of a torus (or the differential unit of volume of a  $S^1 \times P^2$ ), in particular explaining the term  $(1/(4\pi^2))^2$ .

### III. CONCLUSIONS

This collection of work represents attempts to calculate the fine structure constant using the assumptions of knot physics. While some of the results are suggestive, nothing is conclusive. The author intends to continue addressing the problem. However, it seems to be in the best interest of the theory to offer the current progress and invite anyone interested to participate.

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[1] C. Ellgen [www.knotphysics.net](http://www.knotphysics.net) Knot physics, spacetime in co-dimension 2